UNSTEADY HEAT TRANSFER BETWEEN A DENSE SLAB OF DISPERSE MATERIAL AND AN OBJECT IN IT WITH BOUNDARY CONDITIONS OF THE FOURTH KIND

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A solution is found for the unsteady problem of heat transfer between a dense slab of disperse material and an object within this slab for the case of boundary conditions of the fourth kind. The calculated results are compared with experiment.

In studies of unsteady heat transfer between a dense slab of a disperse material and an object within the slab, the temperature at the surface of the object is usually assumed to be constant over time (see, e.g., [1-4]).

In several situations, however, heating or cooling of the object in the disperse medium is significant; then the calculations of the heat fluxes and temperatures of the object must be carried out by jointly solving the heat-conduction equations for the medium and the object. Obviously, the temperatures and heat fluxes must be equal at the boundary between the object and the medium (these are boundary conditions of the fourth kind). A problem of this type was treated in [5], but for only for large values of the Fourier number Fo. An experimental study was carried out in [6, 7]. In the parameter ranges studied, however, it was not possible to determine how the rate of heat transfer was affected by the varying surface temperature. Numerical calculations have also been carried out [7]; they have shown that in the case of boundary conditions of the fourth kind, in contrast with the case of boundary conditions of the first kind, it is necessary to introduce at least one more parameter, to take into account the ratio of the specific heats at constant volume of the medium and the object.

We restrict the present analysis to heat transfer of thin objects; i.e., we neglect the temperature changes over the cross section of the object. We also assume that the temperature drops in the slab and the temperature level are small, so that we can neglect radiation, and we assume that all the thermal properties are independent of the temperature.

We consider the heat exchange between an object immersed in the bed and the disperse medium. The object is a plate of thickness δ, with a thermal conductivity so high that we can neglect the temperature drop over its cross section.

To solve this problem we use the model developed in [9]. The hyperbolic heat-conduction equation for the one-dimensional case can be derived from two equations (see, e.g., [8]):

$$q = -\lambda \frac{\partial \vartheta}{\partial X} - \tau_r \frac{\partial q}{\partial \tau}, \tag{1}$$

$$C\rho \frac{\partial \vartheta}{\partial \tau} = -\frac{\partial q}{\partial X}. \tag{2}$$

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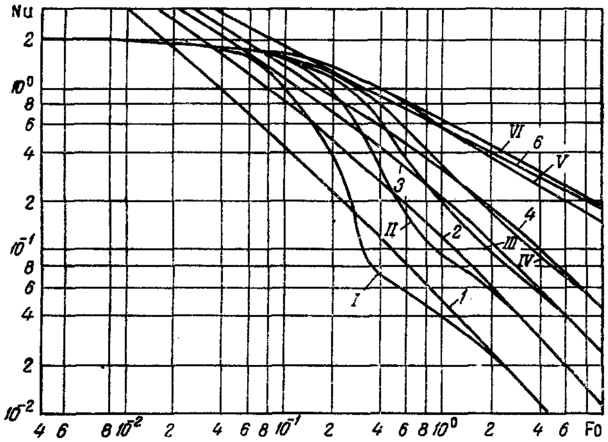

Fig. 1. Calculated dependence of the dimensionless heat flux on the dimensionless time. I-VI) Solutions of the hyperbolic equation for the disperse system for $\mu = 0.1$; 0.25; 0.5; 1; 10; and 1000, respectively; 1, 2, 3, 4, 6) solutions of the classical heat-conduction equation for a single-phase system for $\mu = 0.1$; 0.25; 0.5; 1; 10; and 1000, respectively.

Equation (1) is a generalized heat-transfer law incorporating relaxation, while Eq. (2) is the heat-balance equation.

We adopt the following boundary conditions for a semiinfinite medium:

$$\vartheta(X,\ 0) = 0,\ X > 0, \tag{3}$$

$$q(X,\ 0) = 0,\ X > 0, \tag{4}$$

$$\vartheta(0,\ 0) = \vartheta_0, \tag{5}$$

$$C_0\rho_0\delta \left.\frac{\partial\vartheta}{\partial\tau}\right|_{X=0} = q|_{X=0}\ . \tag{6}$$

Equations (5) and (6) give the conditions of contact and heat exchange between the slab of disperse material and the plate.

In terms of dimensionless variables, problem (1)-(6) is

$$\mathrm{Nu} = -\frac{\partial\Theta}{\partial Y} - \mathrm{Fo}_r\frac{\partial\,\mathrm{Nu}}{\partial\,\mathrm{Fo}}, \tag{7}$$

$$\frac{\partial\Theta}{\partial\,\mathrm{Fo}} = -\frac{\partial\,\mathrm{Nu}}{\partial Y}, \tag{8}$$

$$\Theta(0,\ Y) = 0,\ Y > 0, \tag{9}$$

$$\Theta(0,\ 0) = 1, \tag{10}$$

$$\mathrm{Nu}(0,\ Y) = 0,\ Y > 0, \tag{11}$$

$$-\mu\left.\frac{\partial\Theta}{\partial\,\mathrm{Fo}}\right|_{Y=0} = \mathrm{Nu}|_{Y=0}. \tag{12}$$

Using the method of integral Laplace transforms, we find a solution of Eqs. (7)-(12) for the heat flux at the slab–plate interface:

$$\mathrm{Nu}(0,\ \mathrm{Fo}) = \mathrm{Fo}_r^{-1/2}\exp\left(-\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right)I_0\left(\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right) + \exp\left(-\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right)\mu^{-2}\mathrm{Fo}^{-3/2}\left(\frac{1}{4\mathrm{Fo}_r^2} + \frac{1}{\mu^2\mathrm{Fo}_r}\right)^{-1/2}$$

$$\times\int_0^{\mathrm{Fo}} I_0\left(\frac{\xi}{2\mathrm{Fo}_r}\right)\mathrm{sh}\left[\sqrt{\frac{1}{4\mathrm{Fo}_r^2} + \frac{1}{\mu^2\mathrm{Fo}_r}}\,(\mathrm{Fo}-\xi)\right]d\xi$$

$$-\frac{1}{\mu\mathrm{Fo}_r}\exp\left(-\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right)\mathrm{sh}\left[\sqrt{\frac{1}{4\mathrm{Fo}_r^2} + \frac{1}{\mu^2\mathrm{Fo}_r}}\right]\left(\frac{1}{4\mathrm{Fo}_r^2} + \frac{1}{\mu^2\mathrm{Fo}_r}\right)^{-1/2}. \tag{13}$$

A check of Eq. (13) for certain particular cases verifies the solution. Specifically, we find Nu(0, 0) $= \mathrm{Fo}_r^{-1/2}$, which agrees with the data of [9]. In the limit $\mu \to \infty$ Eq. (13) converts into an equation derived in [9] for boundary conditions of the first kind:

$$\mathrm{Nu}(0,\ \mathrm{Fo}) = \mathrm{Fo}_r^{-1/2}\exp\left(-\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right)I_0\left(\frac{\mathrm{Fo}}{2\mathrm{Fo}_r}\right). \tag{14}$$

The equations for $\Theta(0,\ \mathrm{Fo})$ and Nu(0, Fo) for large values of Fo were found by taking the corresponding limits in transform space; they are

$$\mathrm{Nu}(0,\ \mathrm{Fo}) = (\pi\mathrm{Fo})^{-1/2} - \mu^{-1}\exp\left(\frac{\mathrm{Fo}}{\mu^2}\right)\mathrm{erfc}\left(\frac{\sqrt{\mathrm{Fo}}}{\mu}\right), \tag{15}$$

$$\Theta(0,\ \mathrm{Fo}) = \exp\left(\frac{\mathrm{Fo}}{\mu^2}\right)\mathrm{erfc}\left(\frac{\sqrt{\mathrm{Fo}}}{\mu}\right),$$

$$\tilde{\mathrm{Nu}} = \frac{\mathrm{Nu}}{\Theta} = (\pi\mathrm{Fo})^{-1/2}\exp\left(-\frac{\mathrm{Fo}}{\mu^2}\right)\left[\mathrm{erfc}\left(\frac{\sqrt{\mathrm{Fo}}}{\mu}\right)\right]^{-1} - \frac{1}{\mu}. \tag{16}$$

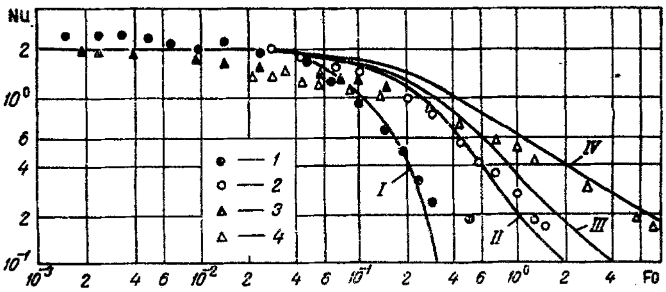

Fig. 2. Calculated and experimental data on unsteady heat exchange at a plate in a dense bed of disperse material. 1-4) Experimental data for μ = 0.1; 0.5; 0.7, and 3.6, respectively; I-IV) solution of the hyperbolic equation for a disperse medium with μ = 0.1; 0.5; 1, and 10, respectively.

TABLE 1. Values of the Parameter μ and Geometric Dimensions of the Particles and the Wall

d, mm	δ, mm	μ
0,93	0,32	0,5
5,05	0,32	0,1
0,93	2,18	3,56
5,05	2,18	0,66

Equation (16) is the familiar solution for a plate in contact with a homogeneous semiinfinite medium [10].

It was shown previously [9] that the solutions of the hyperbolic equation in the case of boundary conditions of the first kind for the temperature and the heat flux at the boundary are the same as the solution of the system of heat-conduction equations for a disperse medium [11]. In terms of the present notation, and taking into account the smallness of the term with the derivative of the gas temperature with respect to the time, we can write this system of equations and the boundary conditions of the fourth kind as follows:

$$\mathrm{Fo}_r \frac{\partial \Theta_1}{\partial \mathrm{Fo}} = \Theta_2 - \Theta_1, \tag{17}$$

$$\mathrm{Fo}_r \frac{\partial^2 \Theta_2}{\partial Y^2} = \Theta_2 - \Theta_1, \tag{18}$$

$$\Theta_1(0,\ Y) = \Theta_2(0,\ Y) = 0,\ Y > 0, \tag{19}$$

$$\Theta_2(0,\ 0) = 1, \tag{20}$$

$$-\mu \left.\frac{\partial \Theta_2}{\partial \mathrm{Fo}}\right|_{Y=0} = \mathrm{Nu}|_{Y=0}. \tag{21}$$

Using the method of integral transforms we can easily show that the solution of Eqs. (17)-(21) for $\Theta(0, \mathrm{Fo})$ and Nu(0, Fo) is the same as the solution of hyperbolic equations (7), (8). Accordingly, to calculate $\widetilde{\mathrm{Nu}}(0, \mathrm{Fo})$ we use, not (13), but the program worked out previously for a numerical solution of system (17)-(21). The results of these calculations, for which we assumed $\mathrm{Fo}_r = 0.25$ [9], are shown in Fig. 1; we see that the dimensionless heat flux under boundary conditions of the fourth kind, even when referred to the instantaneous reduced temperature difference, is smaller than the heat flux in the case of a constant temperature at the boundary. Also shown in this figure are the corresponding curves obtained through a solution of the ordinary differential heat-conduction equation with boundary conditions of the fourth kind, with the disperse medium treated as a homogeneous medium with certain effective properties. At the same time, the heat flux for certain values of Fo is larger (and for certain values it is smaller) than the heat flux in a homogeneous medium. These features of heat exchange at an object in a disperse medium, incorporating a temperature change at the surface of the object, must be taken into account in calculating heat exchange in both a dense bed and in a fluidized bed. In the latter case, this circumstance can turn out to be important, if the change in the surface temperature during a unit contact with the dense phase is quite large. It is interesting to compare these results with the experimental data. The experiments of [6] were carried out for $0.3 \le \mu \le 3.9$, Fo < 5. We see from the calculated curves in Fig. 1 that the curves for different values of Fo > $5 \cdot 10^{-2}$ become distinct at μ. In this range, experiments were carried out for d = 0.39 and 0.93 mm and for δ = 1 mm, corresponding to μ = 3.9 and μ = 1.65. At these values of μ, the influence of this parameter is slight. We therefore carry out experiments with copper plates of two dimensions and in beds consisting of glass spheres of two sizes (Table 1). The experimental apparatus is a rectangular column with Plexiglas walls 7 mm thick and a cross section of 175 × 48 mm. As the calorimeter we use plates 100 × 50 × 0.32 mm and 100 × 50 × 2.18 mm in size. On one surface of the plate, in a groove provided for the purpose, there is a copper-wire heater 0.10 mm in diameter while on the other surface there is a resistance thermometer (copper wire, 0.02 mm in diameter). The calorimeter is placed along the axis of the large sides of the column at the lower part of the apparatus. In the experiments, the disperse material in the column is in contact with a preheated calorimeter, and the cooling process is monitored.

The experimental procedure and the procedure for analyzing the experimental data are described, along with the schematic circuit of the electrical measurements, in [4, 12].

The experimental results are compared with the calculated curves in Fig. 2. We see that the experimental and calculated data agree satisfactorily, justifying the use of the present model for describing heat transfer in a disperse medium under boundary conditions of the fourth kind. These results also show that the influence of the parameter μ must be taken into account in a study of heat exchange of objects immersed in a bed.

NOTATION

q	is the heat flux;
$C_0\rho_0$	is the volume specific heat of wall;
$C\rho(1-\varepsilon)$	is the volume specific heat of disperse system;
X	is the coordinate;
$Y = X/d$	is the dimensionless coordinate;
d	is the particle diameter;
ε	is the porosity;
ϑ	is the temperature;
ξ	is the integration variable;
Θ	is the dimensionless temperature;
τ	is the time;
τ_r	is the relaxation time;
α	is the heat-transfer coefficient for the slab of disperse material and the surface;
λ	is the thermal conductivity;
$Nu = \alpha d/\lambda$	is the Nusselt number;
$Fo = \lambda\tau/C\rho(1-\varepsilon)d^2$	is the Fourier number;
Fo_r	is the dimensionless relaxation time;
δ	is the plate thickness;
$\mu = C_0\rho_0\delta/2C\rho(1-\varepsilon)d$	is the dimensionless parameter.

Indices

1	is the solid phase.
2	is the gas phase.

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